

(10/24/01 The reflected light field and intensity were missing a factor of t_r^* , and prelab question 3 has been slightly modified.)

Optical Resonators and Mode Matching

Abstract

This laboratory experiment explores the characteristics of an optical resonator excited by an external laser beam. The first portion of the experiment seeks to provide some experience in aligning a set of mirrors to form a resonator such that the input laser beam is reasonably well-coupled into the system. One of the resonator mirrors is scanned back and forth using a piezo element so that the length of the resonator changes in a periodic manner. An array of different spatial modes will be observed as the length is changed slowly. The second portion of the experiment provides some experience with mode matching of the laser beam to the mode of the resonator. Proper mode matching largely eliminates the excitation of all but the fundamental Gaussian mode. Once the resonator is mode matched, various resonator parameters will be assessed, such as cavity decay time, finesse, linewidth, and so on.

References:

Fundamentals of Photonics by Bahaa E. A. Saleh and Malvin C. Teich, John Wiley and Sons, New York, (1991):

Chapter 1 section 4 on ray matrices.

Chapter 3 on Gaussian beams.

Chapter 9 on resonators.

Note: The notation used here is primarily that of Saleh and Teich, except here the symbol “ i ” is used for the $\sqrt{-1}$ instead of j . To convert to the notation of the text, replace the i ’s here by $-j$.

1 Introduction

A set of two or more mirrors arranged to cause light to propagate in a closed path is variously called an optical resonator, an optical cavity, and in certain contexts, an optical interferometer. Optical resonator along with optical gain are the basic elements of every laser, optical resonators are used in sensors such as the laser gyroscope, and they are used extensively in optical precision measurements, to name merely a few applications. Indeed the optical resonator plays a ubiquitous role in modern optics and it is for this reason that it is chosen as a subject of study in the Advanced Optics Laboratory.

This laboratory experiment is designed to explore the properties of an optical resonator; properties that are common to nearly every optical resonator. Furthermore it is designed to provide some experience in aligning an optical resonator and with getting laser light *into* an optical resonator—a procedure known as *mode matching*.

There are two “tricky” parts to this lab. First, aligning a resonator to a laser beam is non trivial at first. Over time an optics expert simply becomes good at it. Second, as is discussed below, the process of mode matching a laser beam to a resonator is also not trivial for first-time students. The main reason is that there is no unique solution to the problem—there are many

possible ways of solving the problem yet still many more ways to *not* solve the problem. Mode matching is thus somewhat of an art, although calculations along the way are essential. Do not be discouraged, for it is exactly this Art of resonators that this laboratory exercise gives you a chance to develop.

2 Optical resonators

There are two basic types of optical resonators: the standing wave resonator and the traveling wave, or ring, resonator. The simplest resonator consists of two facing mirrors, of which at least one is usually spherical. A ring resonator consists of at least three mirrors. In this experiment you will be constructing a four-mirror traveling-wave resonator.

2.1 RESONATOR MIRRORS

Typically resonators are formed of a set of physically distinct mirrors, although there are common exceptions. The reflectors of a diode laser are simply the cleaved faces of the semiconductor material. The index-of-refraction of the semiconductor is so high compared to the surrounding air that it reflects a substantial portion of the incident light. Nowadays, total-internal reflection within a sphere or other geometrical structure is also often utilized in optical resonators.

In the visible and infrared region of the optical spectrum mirrors are formed either of metal coatings (primarily aluminum, gold, or silver) or of many layers of dielectric materials (for example, alternating layers of silicon dioxide and titanium dioxide).

Any given mirror may be characterized by its field amplitude reflection coefficient r and its transmission coefficient t . In general these are complex-valued quantities. Typically we are interested in the *intensity* reflectivity $R \equiv |r|^2$ and transmission $T \equiv |t|^2$. No mirror is perfect: the mirror material absorbs some amount of the incident light. Furthermore, the surface of the mirror is also always imperfect and scatters some amount of light. At the very least, the surface of a mirror has the roughness on the scale of the size of that atoms that make up the mirror surface. Whether light is scattered or absorbed, for our purposes it is simply lost. We can lump the energy loss in single coefficient L .

Energy conservation dictates that the sum of the light intensity reflection, transmission, and loss is unity:

$$R + T + L = 1. \tag{1}$$

It may seem surprising that the very highest mirror reflectivities are obtained with dielectric mirrors. The losses and the transmission of good dielectric mirrors is often given in ppm, i.e., parts-per-million. The very best mirrors have a loss of under a few parts per million. The corresponding reflectivity might be 0.999998. The surface irregularities of such fantastic mirrors is nearly at the atomic scale!

2.2 RESONATOR FIELDS

Consider the simplest two-mirror resonator having a monochromatic laser beam incident on the back of one *input* mirror. Of interest in this section is the intensity of light within the

resonator, the intensity of light exiting through the output mirror, and the intensity of light reflected from the resonator.

A most remarkable and useful fact is that the light intensity within the resonator can be hundreds to million times higher than the incident light intensity. Furthermore, the light reflected from the resonator can be nearly zero despite the fact that the input mirror may have very high reflectivity. At the same time, nearly all of the incident light is transmitted through the output mirror despite the possibility that its transmission is very low. The goal here is to understand these useful facts.

Let the input mirror coefficients be r_1 and t_1 and the output mirror coefficients be r_2 and t_2 . It will sometimes be convenient to write these in polar form, such as $t_1 = |t_1|e^{i\phi_1} = \sqrt{T_1}e^{i\phi_1}$. The input frequency we take to be ν and the wave has corresponding free-space wavenumber $k = 2\pi / \lambda = 2\pi\nu / c$, where c is the speed of light. If the input field amplitude of the light is E_0 , then just to the right of the input mirror the field amplitude is:

$$E_{i_0} = t_1 E_0. \quad (2)$$

This wave that has just entered the resonator then propagates down to the output mirror and a portion of it reflects and travels back to the input mirror where another portion of it reflects again. Having made a complete round trip this original wave then has a resulting amplitude:

$$E_{i_1} = t_1 E_0 r_2 r_1 e^{2ikd}, \quad (3)$$

where d is the separation between the mirrors. This latter field then makes another round trip in the resonator, whence it becomes:

$$E_{i_2} = t_1 E_0 (r_1 r_2)^2 e^{4ikd}. \quad (4)$$

Basically the same thing happens to the wave with each round trip so that after n round trips the field becomes

$$E_{i_n} = t_1 E_0 (r_1 r_2)^n e^{2nikd}. \quad (5)$$

If the input field has remained present for the entire time, then in reality the field just to the right of the input mirror is a sum of waves that have made no round-trips, one round-trip, etc. The total field can thus be written as a series that has a well known sum:

$$\begin{aligned} E_i &= t_1 E_0 \sum_{n=0}^{\infty} (r_1 r_2)^n e^{2inikd} \\ &= \frac{t_1 E_0}{(1 - r_1 r_2 e^{2ikd})} \end{aligned} \quad (6)$$

The corresponding intensity internal to the resonator is proportional to:

$$I_i = |E_i|^2 = \frac{T_1}{|1 - \sqrt{R_1 R_2} e^{(2ikd + i\phi_0)}|^2} I_0, \quad (7)$$

where

$$\phi_0 = \arctan\left(\frac{\text{Im}\{r_1 r_2\}}{\text{Re}\{r_1 r_2\}}\right). \quad (8)$$

The transmitted field and intensity is easy to calculate from the internal field:

$$E_t = \frac{t_1 t_2 e^{ikd}}{(1 - r_1 r_2 e^{2ikd})} E_0, \quad (9)$$

$$I_t = \frac{T_1 T_2}{|1 - \sqrt{R_1 R_2} e^{(2ikd + i\phi_0)}|^2} I_0. \quad (10)$$

The field reflected from the resonator has two contributions: one from the incident field directly reflected from the input mirror and the other transmitted back though the input mirror from inside the resonator.

$$E_r = \left[-r_1^* + \frac{r_2 T_1 e^{2ikd}}{(1 - r_1 r_2 e^{2ikd})} \right] E_0. \quad (11)$$

The fact that the directly reflected light is has a $-r_1^*$ rather than simply r_1 arises from imposing energy conservation constraints on a single mirror –a manifestation of something called the *Stokes relations*. The corresponding reflected intensity is rather simpler than it looks:

$$I_r = \frac{\left| -1 + \left(1 + \frac{T_1}{R_1} \right) \sqrt{R_1 R_2} e^{2ikd + i\phi_0} \right|^2}{|1 - \sqrt{R_1 R_2} e^{2ikd + i\phi_0}|^2} R_1 I_0, \quad (12)$$

2.3 RESONANCE

The three intensities of interest share a common denominator. Values of the input frequency that satisfy $2kd + \phi_0 = 2n\pi$ or $2d\nu / c = n$ where n is an integer are called *resonant frequencies*. At these frequencies the denominator becomes small and the internal and transmitted intensities both become large while the reflected intensity can become small.

2.4 FREE SPECTRAL RANGE

The free spectral range is the frequency spacing between resonances given by n and $n+1$:

$$FSR = \frac{c}{2d}.$$

Since $2d$ is the *round-trip* path length one can simply remember that the free spectral range is simply the speed of light divided by the round-trip path length, or simpler yet, it is the inverse of the round-trip travel time.

2.5 RESONANCE CHARACTERISTICS

The resonance characteristics of an optical resonator are characterized in several more or less equivalent ways, the choice of which depends upon the emphasis and application.

2.5.1 Bounce number

The effective number of round-trips a photon makes before it has a 1/e probability of escaping the resonator by transmission or loss mechanisms.

$$b = 1 / \text{Losses}, \quad (13)$$

where the Losses are the sum of *all* resonator intensity losses including mirror losses, mirror transmissions, and anything else that might produce loss in the resonator. For the simple two-mirror resonator $\text{Losses} = T_1 + L_1 + T_2 + L_2 = 2 - R_1 - R_2$.

In general a large bounce number is often desirable.

2.5.2 Cavity decay time

Also called the photon lifetime. It is simply given by $t_c = b / \text{FSR} = 2db / c$. Note that this quantity is independent of resonance. Once a photon enters a cavity it does not matter whether its frequency is or is not at a resonance, it still has the same lifetime in the resonator.

2.5.3 Linewidth

For a resonator with low losses (less than about 1%) the line shape is essentially Lorentzian. The full width at half maximum, or FWHM, is called the linewidth $\delta\nu$.

$$\delta\nu = \frac{\text{FSR}}{2\pi b}. \quad (14)$$

In general a small linewidth is desirable.

2.5.4 Cavity finesse

For a two-mirror cavity the cavity finesse is defined by:

$$F = \frac{\pi(R_1 R_2)^{1/4}}{1 - \sqrt{R_1 R_2}}. \quad (15)$$

To the extent that the cavity losses are low it is also given by: $F = 2\pi b$, which is trivial to calculate and is valid no matter what the geometry of the resonator happens to be. In many respects the bounce number supplants the cavity finesse in modern applications of resonators.

Many of the resonator characteristics that quantify the performance of the resonator are dependent on its length. An example is the linewidth given above. The longer the cavity, the narrower its linewidth. The finesse is independent of the cavity length and therefore is indicative of the quality of the mirrors alone.

2.5.5 Cavity Buildup

The intensity inside the resonator on resonance is simply:

$$I = bI_0, \quad (16)$$

provided the losses are low.

2.5.6 Quality factor

The quality factor of a resonator, or *cavity* Q , is a measure of the resonator's ability to store energy. Resonator Q is often used to characterize electronic circuits and microwave elements as well. For an optical resonator:

$$Q = \nu / \delta\nu. \quad (17)$$

A high Q is indicative of both high energy storage capability and in general a narrow linewidth. In optics the quality factor is not used terribly much except to make a comparison with analogous elements in the electronics domain.

2.5.7 Impedance matching

When the mirror parameters are such that on resonance *no light is reflected from the resonator* (or apparently no light reflected from the input mirror), the resonator is said to be *impedance matched*. The condition for impedance matching is simple: the input mirror intensity transmission T_i must equal the sum of all other losses in the resonator, i.e., including the sum of transmissions of all of the other mirrors of the resonator as well as the total absorption and scattering losses of all elements of the resonator. This assumes that the losses and transmissions are all small so that terms or second order and higher in losses may be neglected.

2.6 EXAMPLE:

Consider a high quality cavity that employs two identical mirrors having 10 ppm loss and 90 ppm transmission. The bounce number is then $b = 1 / (2 \cdot 1 \times 10^{-4}) = 5000$ and the finesse is about 31,000 (very high). If the incident intensity is 1 mW, then the internal intensity is 5 W. Say the mirror separation is 30 cm. Then the free spectral range is $FSR = 3 \times 10^8 / (2 \cdot 0.3) \text{ Hz} = 2.2 \text{ GHz}$. The decay time is then $\tau_c = 10 \mu\text{s}$ and the linewidth (FWHM) is $\delta\nu = 16 \text{ kHz}$.

3 Resonator Spatial Modes

The field summation done above to find the internal, reflected, and transmitted fields had an implicit assumption, namely, that the fields are properly described by plane waves. In reality the spatial distribution of the fields of a resonator depends on the shape of its mirrors. A given resonator generally supports modes having a well-defined and discrete spatial structure. For the case of resonators constructed purely of spherical elements (including planar) the modes are described by *Hermite-Gaussian* beams.

3.1 FUNDAMENTAL MODE

The most basic mode of a spherical-mirrored resonator is the Gaussian mode. Any cross section of a Gaussian beam by definition has a transverse electric field distribution that follows a Gaussian profile. As a beam propagates, however, its wavefront changes. The complete Gaussian beam is a bit complicated. Let the beam propagation direction be z and the transverse radial coordinate be designated with ρ . Then the beam amplitude can be written:

$$U(\vec{r}) = E_0 \frac{W_0}{W(z)} \exp\left[-\frac{\rho^2}{W^2(z)}\right] \exp\left[ikz + ik\frac{\rho^2}{2R(z)} - i\zeta(z)\right], \quad (18)$$

where the transverse beam size is characterized by a size:

$$W(z) = W_0 \left[1 + \left(\frac{z_0}{z}\right)^2\right]^{1/2}, \quad (19)$$

and the wavefront curvature evolves as:

$$R(z) = z \left[1 + \left(\frac{z_0}{z}\right)^2\right]. \quad (20)$$

The remaining parameters are the phase

$$\zeta(z) = \arctan \frac{z}{z_0}, \quad (21)$$

and the waist size

$$W_0 = \left(\frac{\lambda z_0}{\pi}\right)^{1/2}. \quad (22)$$

The parameter z_0 is called the *Raleigh range*. It and the location of the origin $z=0$ are determined by boundary conditions, to which we will soon return.

3.1.1 Intensity profile

The intensity profile along any given cross section of the beam is also Gaussian:

$$I(\rho, z) = I_0 \left[\frac{W_0}{W(z)}\right]^2 \exp\left[-\frac{2\rho^2}{W^2(z)}\right]. \quad (23)$$

Evidently the beam is characterized by a radius W , which is a function of position along the beam propagation direction. At a radius of $\rho = W$ the beam intensity is down by a factor of $1/e^2$ relative to its peak value. The corresponding diameter $2W$ is called the *spot size*. At $z=0$ the beam has its minimum radius. This value W_0 is called the waist radius and the corresponding diameter is called the waist size. At a position $z = z_0$ the spot size has increased by a factor of $\sqrt{2}$ and continues to increase monotonically. For distances much larger than a Rayleigh range the spot size grows linearly with distance:

$$W(z) \rightarrow W_0 \frac{z}{z_0} = \theta_0 z. \quad (24)$$

The beam divergence can be written:

$$\theta_0 = \frac{\lambda}{\pi W_0}. \quad (25)$$

3.1.2 The q -parameter

The manipulation of gaussian beams with optical elements is sometimes most easily accomplished with the q -parameter, which is a complex-valued parameter that describes both the size and curvature of the beam.

$$\frac{1}{q(z)} = \frac{1}{R(z)} + i \frac{\lambda}{\pi W^2(z)}, \quad (26)$$

in which,

$$q(z) = z - iz_0. \quad (27)$$

3.2 HIGHER-ORDER GAUSSIAN BEAMS

The simple Gaussian mode is just the basic parent of a family of modes. For simple resonators the family of modes are described by an infinite family Hermite-Gaussian functions. One can easily see these *higher-order transverse modes* when the mode matching is poor. They can be quite beautiful! In general the higher-order modes have different resonance frequencies. Therefore as the resonator length is scanned, these modes can be seen to appear in sequence.

4 ABCD matrices

The representation of optical elements for the present purposes is perhaps most easily cast in terms of ABCD matrices. As a reminder, the ABCD matrices for some simple optical elements are given in the table below.

Optical Element	ABCD Matrix
Propagation through a medium having index-of-refraction n and length d	$\begin{bmatrix} 1 & d/n \\ 0 & 1 \end{bmatrix}$
Refraction at a spherical boundary of radius R , entering a medium of index n_2 from a medium of index n_1 . R is positive if the center of curvature lies in the positive direction of ray propagation.	$\begin{bmatrix} 1 & 0 \\ -\frac{n_2 - n_1}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix}$
Transmission through a thin lens of focal length f	$\begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$
Reflection from a spherical mirror having radius R . R is positive if the center of curvature lies in the positive direction of incident ray propagation.	$\begin{bmatrix} 1 & 0 \\ 2/R & 1 \end{bmatrix}$

A complex optical system can simply be expressed as a product of the individual ABCD matrices. Keep in mind that for light propagation from left to right from element to element,

each corresponding ABCD matrix sits on the left of the matrices corresponding to the previous elements.

4.1 MANIPULATION OF A GAUSSIAN BEAM

A lens, mirror, or other optical element generally changes the parameters of a Gaussian beam. One approach to calculating the change is through the ABCD matrix representation of an element or a series of elements. An input q -parameter q_1 is transformed to an output q -parameter q_2 according to:

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D}. \quad (28)$$

This is called the *ABCD law*.

4.2 RESONATORS AND ABCD MATRICES

What is the Gaussian mode that can be associated with a given resonator? The answer is that the ABCD-law must provide a self-consistency between the input and output q -parameter. Start anywhere in the resonator and write down the product of the ABCD matrices (that is, the total ABCD matrix) describing one round trip through the resonator path. Then the q -parameter must satisfy:

$$q = \frac{Aq + B}{Cq + D}. \quad (29)$$

There are two solutions:

$$\frac{1}{q_{\pm}} = \frac{D - A}{2B} \mp \frac{1}{B} \sqrt{\left(\frac{A + D}{2}\right)^2 - 1}. \quad (30)$$

The allowed solution must have a negative imaginary component. Once the q -parameter at a given position inside of the resonator is known, it is simple enough to propagate it to somewhere outside of the resonator, again using the ABCD matrix approach.

5 Mode matching

5.1 LASER MANUFACTURER SPECIFICATIONS

Laser manufacturers typically specify the beam size and beam divergence of their laser beam. However, they often do not actually specify what the waist size or position is. You can use the divergence to calculate the waist size, and you can use the quoted beam size to calculate then calculate the waist position. These two along with the wavelength of the laser emission thus completely characterize the beam for most practical purposes. Even if the numbers do not work out, the mode matching optics can usually be minimally adjusted to improve the matching.

5.2 MODE MATCHING PROCEDURE

At this point all the tools for mode matching are in place. Assuming that the q -parameters of both the laser beam and the resonator beam just outside of the input mirror are known, one only needs to use the ABCD law to perform matching between the two.

The difficulty is that there is not a unique solution to the problem! Many different lens systems can give rise to the same ABCD matrix, and more than one ABCD matrix can produce mode matching. So how to go about the process? Here are some rules of thumb.

If the laser and resonator are in fixed positions, then one can find a solution to the mode-matching condition can (often but not always) be satisfied by lens having a specific focal length placed at a specific location between the laser and resonator. This is rarely a practical approach because one does not usually have just the right focal length lens.

A pair of lenses will often do a great job –several convenient choices for the two lenses can give the focal length of the lens needed above. A good starting point is to choose a ratio of focal lengths to be the ratio of waist sizes between the laser and the resonator.

A third lens allows some additional positioning freedom and can be useful for fine tuning the mode matching.

It is generally a good ideal to avoid both extremely short and extremely long focal lengths. Focal lengths in the range of 20 mm to 500 mm are commonly available.

Mode matching is usually an iterative process. Using a computational aid like Mathematica is very useful for determining a good set of starting lenses from those that one has on hand.

6 Alignment of optical resonators

The first task of this laboratory experiment has you assemble a four mirror resonator and to inject a laser beam into the resonator. Aligning optical resonators is a skill that requires some experience before it can be executed quickly. One perhaps obvious hint is that one should use the small amount of laser light that is transmitted through the input mirror and align the subsequent mirrors so that the round-trip beam intersects itself. A less obvious hint is that only *two* resonator mirrors need to be moved in order to bring the resonator into alignment with the input beam. The input mirror is one good choice to move, and the second mirror might be the one that is furthest away from the input mirror. The mirror adjustments are made in a series of small movements sometimes called “beam walking”. First the horizontal adjustments (say) of the two mirrors are alternately moved and then the vertical adjustments are moved alternately; then one returns to the horizontal, and so on. During the first walking session note the direction chosen to move each mirror. If the alignment deteriorates, then one or both directions have to be reversed. Occasionally one will find that the resonator beam has entirely walked off some mirror, In that case that mirror or another has to be moved and the alignment process is then started anew.

7 Prelab questions

1. Consider a resonator made up of two mirrors having reflectivity $R_1=0.990$, and $R_2=0.995$, $T_1=0.008$, $T_2=0.003$. The mirrors are spaced by 50 cm. The input mirror is curved with radius 2 m (center of curvature towards the back mirror), and the back mirror is flat.

- a) Calculate the Finesse, bounce number, FSR, photon lifetime and linewidth for the resonator.
 - b) Calculate the fundamental mode parameters for this resonator.
 - c) For an input power on the first mirror of 100 mW, calculate the internal, transmitted, and reflected powers.
 - d) Assuming the mirror losses are fixed ($R+T=\text{constant}$ less than 1), what (intensity) transmission would you pick for the second mirror to impedance match the resonator?
2. The laser beam waist size is 0.5 mm and the wavelength is 532 nm. You have a collection of thin lenses having focal lengths anywhere from 1 cm to 500 cm in (approximately) 20% increments (1 cm, 1.2 cm, 1.4 cm, etc.). Design a mode matching system to match the laser to the above resonator. Make a sketch showing the distances from the laser beam waist of the lenses and cavity input mirror.

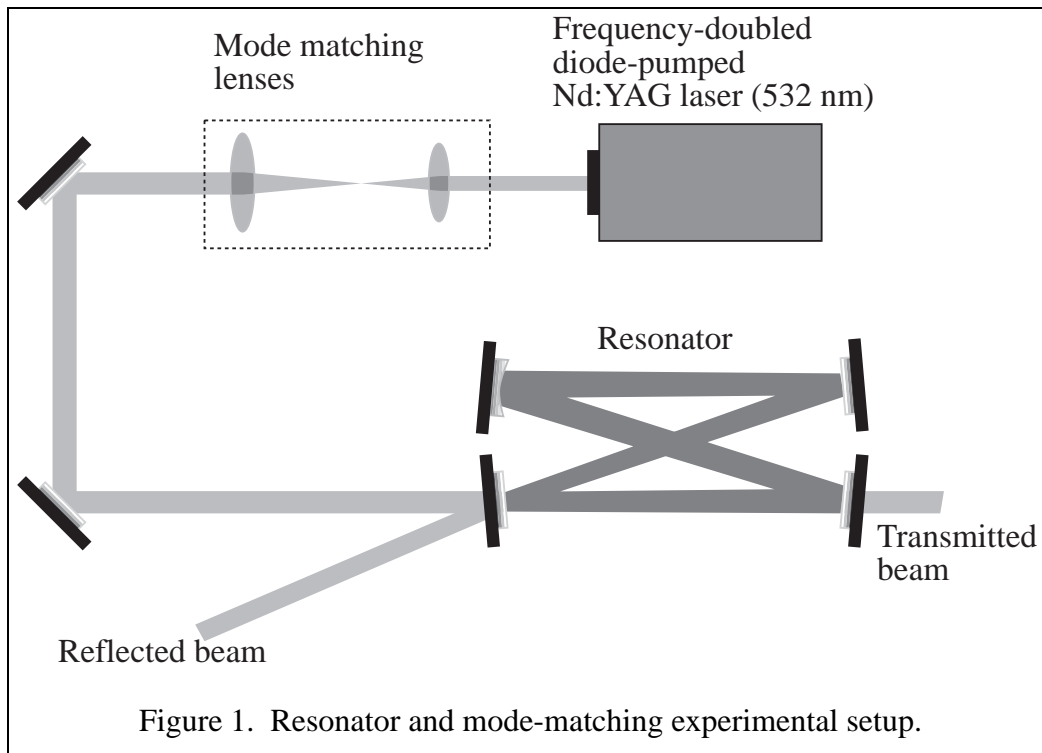
In solving this problem you will want a program to help you calculate the mode-matching condition. Save the program so that you can use it again for the actual resonator that you build in the lab.

3. Show that the impedance matching condition indeed leads to no light reflected from the resonator. To do so, assume that the losses and transmissions are small so that terms of second order in their products are negligible.

8 Procedure

1. Set up the laser and resonator in the configuration shown in Figure 1. Leave out the mode matching lenses. The laser is quite intense. You may want to attenuate the beam. You can do so either by placing an attenuator in its path (make sure you are not burning a hole in the attenuator. You can also attenuate the laser beam by taking its 4% reflection from an uncoated piece of glass.
2. Align the input beam and resonator until the intensity inside the resonator flickers. Notice that patterns on the mirror surfaces.
3. Project the transmitted and reflected intensities on the wall, note the patterns in your lab book. Take some digital photos. How do the patterns change as you misalign and align the resonator?
4. Measure the (uncalibrated) laser power –you will first need to attenuate the laser beam so as not to saturate the detector if you have not already attenuated the beam.
5. Put detectors on the reflected and transmitted beams.
6. Hook the detectors up to a two-channel oscilloscope.
7. One mirror has a piezo element. Hook it up to a signal generator. Put the generator on triangle wave.
8. Note the traces on the oscilloscope. Adjust the amplitude and frequency of the signal generator to get a good trace. (Something between 10 and 100 Hz should be good.)
9. Note how the traces change with alignment/misalignment.
10. Align the resonator to maximize the transmitted intensity.
11. Measure the transmitted and reflected maximum and minimum intensities. Compare with the input intensity.

12. Now design a mode matching system and put it in the beam path (you may have to change the resonator position.) Use the laser data sheet to find the needed beam parameters. Sketch the system you have design showing focal lengths, distances, etc.
13. Attempt to optimize the mode matching by maximizing the peak transmitted intensity. Actually, it is often easier to optimize by minimizing the dip in the reflection trace.
14. Again measure the transmitted and reflected maximum and minimum intensities. Compare with the input intensity. Compare with your previous results.
15. Estimate the total power appearing in all of the higher-order modes, compare with the power in the fundamental mode.



16. Determine the Free Spectral Range from your cavity length.
17. Measure the linewidth using the FSR to calibrate the frequency scale of the oscilloscope. Using the d.c. offset of the signal generator is often helpful.
18. Check your linewidth measurement at various scanning frequencies to see if you obtain consistent results.
19. Calculate the Finesse, photon lifetime, and number of bounces.
20. Compare your results with separate measurements of the transmission of the mirrors and assume the mirrors are lossless to obtain their reflection (measuring reflection directly is difficult).